Addendum

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Introduction

In this paper we will make a brief addendum to the two previously papers *The Feynman Project* and *The Nature of Collisions* with focus on critical values. We have previously calculated the probability of getting a Feynman Point for a normal number and now we will compare it to critical values. Furthermore we want to calculate the probability of getting 48 collisions for a random 10 000 normal number and compare it to critical values. The critical value approach involves determining "likely" or "unlikely" by determining whether or not the observed test statistic is more extreme than would be expected if the null hypothesis were true. That is, it entails comparing the observed test statistic to some extreme values, called the *critical values.* If the test statistic is more extreme than the critical value, then the null hypothesis is rejected in favour of the alternative hypothesis. If the test statistic is not as extreme as the critical value, then the null hypothesis is not rejected. The level of significance α of the test is in this paper predefined to $\alpha = 0.05$.

The Feynman Point compared to critical values

We wish to calculate the probability of the existence of a Feynman Point. We can consider this as a Binomial distribution where

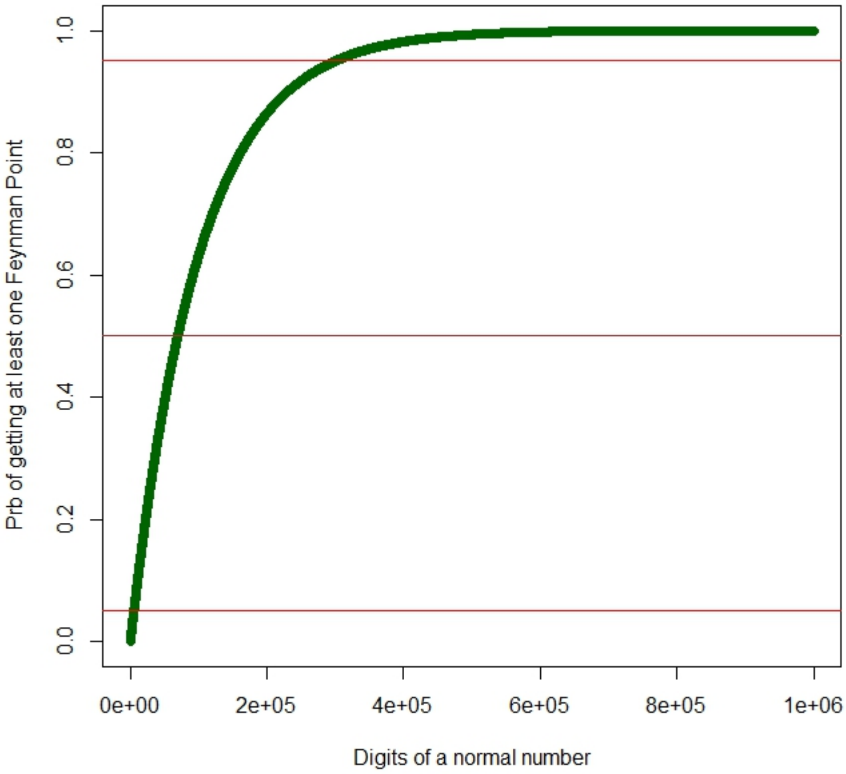
$$P(\text{Number of succes} = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the number of trials and p is the probability of success on the ith trial, where $i \in \{1, ..., n\}$. We clearly have that $p = 10^{-6}$ and n is the length of the sequence we wish to analyse. With these parameters we have that

$$P(k = 0, n, p) = \binom{n}{0} (10^{-6})^0 (1 - 10^{-6})^{n-0} = (1 - 10^{-6})^n$$

is the probability that there are zero Feynman Points in the first n digits of a normal number. Notice that we multiply with 10 since we are looking for ten disjoint Feynman Points $(000000, ..., 999999)$ so that

$$P\,(\text{The probability of getting a Feynman Point}) = 1 - 10^{-5}.$$



We have that

$P(\text{There exist at least one Feynman Point}) = 1 - P(\text{There don't exist a Feynman Point})$

$$= 1 - P(k = 0, n, p) = 1 - \binom{n}{0}(10^{-5})^0(1 - 10^{-5})^{n-0} = 1 - (1 - 10^{-5})^n = \xi$$

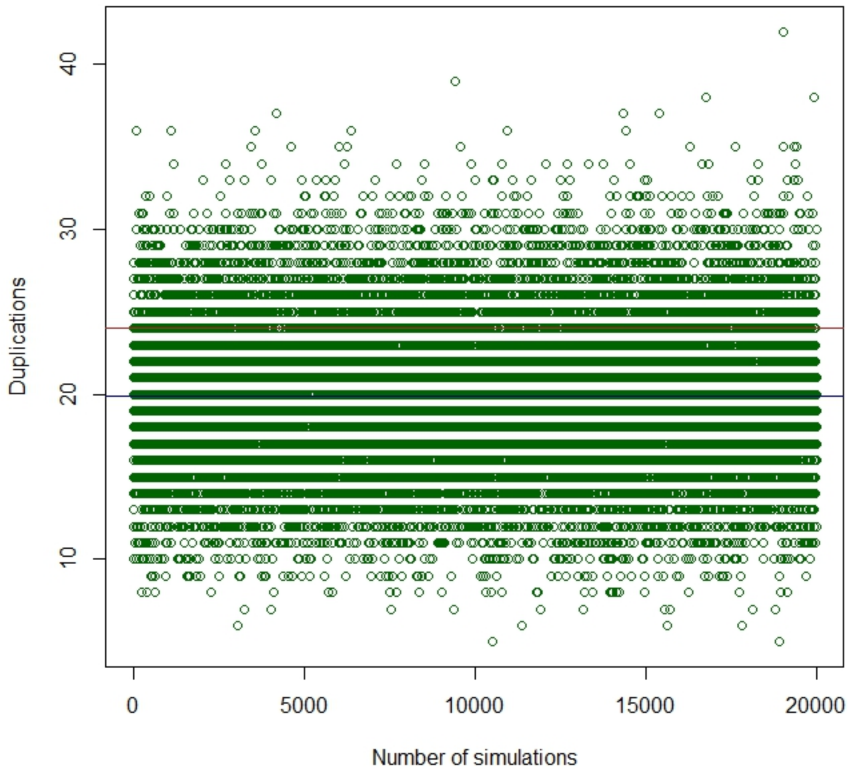
where ξ is a fixed probability. Thereby we get

$$n = \frac{\log(1 - \xi)}{\log(1 - 10^{-5})}.$$

For $\xi = (0.05, 0.5, 0.95)$ we get $5129, 69314, 299572$. By this we see that the Feynman Point in π is clearly a critical value because it appears after digit number 762.

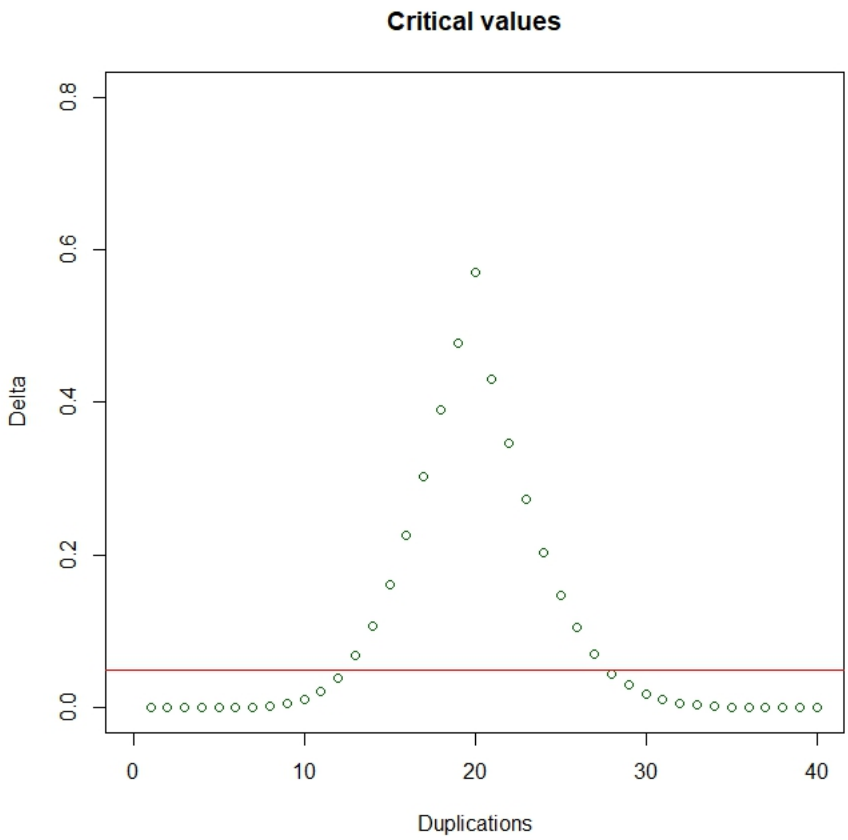
The number of collisions compared to critical values

The first 10 000 digits of π where calculated in 1958 and since that time we knew that we had 48 collisions in π. We will now define *duplications* as a block repeated subtracted so all of the entries in the matrix are unique. So for the first 10 000 digits of π the number of duplications are $\frac{48}{2} = 24$. For a random generated normal number of length 10 000 we investigate the number of duplications after 20 000 simulations.



The probability of getting 24 duplications for a 10 000 generated random normal number is 0.052 and the probability of getting lower is 0.8014 and higher is 0.1466. The mean is close to 20 (19.87) and the minimum value is 5 and maximum value is 42.

The probability δ_i for $i = \{1, 2, ..., 42\}$ is defined as the probability for getting lower a fixed duplication for $i \leq 20$ and the probability for getting higher a fixed duplication for $i > 20$. The values for δ has been calculated in R and the number of duplications for getting δ below 0.05 (critical values) are shown on the graph.



We see that all duplications strictly below 13 or strictly higher than 27 are critical values. Thereby we notice that 24 duplications is not a critical value.

Conclusion

We calculated the interval for non-critical values for the appearance of at least one Feynman Point in π, which is $[5129 : 299572]$. Since the first Feynman Point in π appears after digits number 762, we can conclude that it is a critical value. Similarly we calculated the interval for non-critical values for the number of duplications in the first 10 000 digits of π, which is $[13 : 27]$. Since we have 24 duplications (which is 48 collisions in our case) in the first 10 000 digits of π, we can conclude that it is not a critical value.

Appendix

The R Code

```
# Reading data
p=read.table("C:\\Users\\Mark\\Desktop\\data10000.txt",
header=TRUE)

# Splitting the data into lines
text =
   readLines("C:\\Users\\Mark\\Desktop\\data10000.txt"
,encoding="UTF-8")

v=c(strsplit(text,"")[[1]])
# convert string v to integer v
strtoi(v, base = 10)

# Chose the number of blocks
h=2000

u=matrix(v,ncol=5,nrow=h,byrow=TRUE)


#### Feynman Project
# Multiplied by 10.
b=c()
for(j in 1:10^6){
# Prb that there is at least one Feynman Point
b[j]= 1- pbinom(0,size=j,prob=1/10^5)
}
plot(b,col="darkgreen",main="",xlab="Digits of a normal
   number",
ylab="Prb of getting at least one Feynman Point")
abline(h=0.5,col="brown")
abline(h=0.95,col="red")
abline(h=0.05,col="red")

#####
# xi
xi=c(0.05,0.5,0.95)
log(1-xi)/log(1-10^(-5))

## Collisions
# Generate 10 000 random number and return the number of
   collisions
# save them in a vector co
co=c()
#number of simulations
m=20000
for(j in 1:m){
        v=sample(0:9,size=10000,replace=TRUE)#generate
            10000 random digits
```

```
        u_sim=matrix(v,ncol=5,nrow=2000 ) # make them as
           a matrix
        # Remove all duplications (not collisions) by
           using the unique function.
      # and subtract it from the number of blocks.
        co[j]=2000-dim(unique(u_sim))[1]
}
# Thum of rule. Times by two if all are double
   duplications.
# Note there may exist tripel duplications, however, we
   only look at the
# number of duplications.

plot(co,col="darkgreen",ylab="Probability for getting x
   duplication")
abline(h=mean(co),col="darkblue")
abline(h=24,col="brown")
# The brown line is the number of duplications in pi
# The blue line is the mean of all duplications

# Number of duplications above 24
length(co[co==24])/m
length(co[co>24])/m
length(co[co<24])/m
c( mean(co),min(co),max(co))
#mean is close to 20,min is close to 4,max is close to 40

# Find critical low values
crit_low=c()
for(k in 1:20) {
crit_low[k]=length(co[co<=k])/m
}
plot(crit_low,col="darkgreen",ylab="Delta",
xlab="Duplications",main="Critical
   values",xlim=c(0,40),ylim=c(0,0.8))

# Find critical values for high values
crit_high=c()
for(k in 21:40) {
crit_high[k]=length(co[co>=k])/m
}
points(crit_high,col="darkgreen")
abline(h=0.05,col="red")
```

References

http://www.worldpifederation.org